

STUDY ON DIFFERENT HEDONIC GAMES AND THEORIES OF STABILITY



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ABSTRACT

One of the results of this kind of game is a partition of the player set, also known as a coalition structure. This refers to a collection of coalitions, whose union is equal to the set of players, but which cannot be joined pairwise with one another. Marriage difficulties and roommate problems (Gale and Shapley (1962), Roth and Sotomayor (1990)) can both be viewed as special examples of hedonic coalition formation games. In these games, each agent solely considers who will be his or her partner. Problems can arise in both of these situations. In point of fact, hedonic games are only condensed versions of more

general games involving the creation of coalitions. In these games, the manner in which a coalition's total payout is to be distributed among its members is predetermined and made transparent to all agents. Hedonic coalition formation games are a powerful tool to model agent behavior in a variety of circumstances. A classic example is the stable marriage problem which seeks to pair off men and women from two equally-sized groups. In the stable marriage problem, a pair is said to be stable if there is no man-woman pair who both want to leave their assigned partners to form a new pair.

Keywords: Hedonic, Games, Theories

INTRODUCTION

When confronted with particular economic and political conditions, such as the provision of public goods in local communities or the formation of clubs and organizations, individuals take action by banding together to create coalitions. Modeling such a setting as a (strictly) hedonistic coalition building game is one approach to describing the environment in question.

In a hedonic coalition formation game, there is a finite and non-empty set of players, as well as a list of preferences for those individuals, with the expectation being that the preferences of each player are solely determined by the other players in her coalition. One of the results of this kind of game is a partition of the player set, also known as a coalition structure. This refers to a collection of coalitions, whose union is equal to the set of players, but which cannot be joined pairwise with one another. Marriage difficulties and roommate problems (Gale and Shapley (1962), Roth and Sotomayor (1990)) can both be viewed as special examples of hedonic coalition formation games. In these games, each agent solely considers who will be his or her partner. Problems can arise in both of these situations. In point of fact, hedonic games are only condensed versions of more general games involving the creation of coalitions. In these games, the manner in which a coalition's total payout is to be distributed among its members is predetermined and made transparent to all agents.

A partition is said to have a core that is stable if there is no coalition that has members who would prefer to belong to that coalition rather than the coalition to which they now belong under the given partition. A partition is said to be Nash stable if there is no player who would benefit from leaving her current coalition to join another coalition of the partition, which could be referred to as the "empty coalition" in this context. If there is such a player, then the partition does not have Nash stability. Take note that a core stable partition does not necessarily have to be Nash stable, and that a Nash stable partition does not necessarily have to be core stable.

When contrasting or comparing different types of stability, it is important to keep in mind two important aspects: (i) who can deviate from the given partition (for example, a coalition of players in core stability, a singleton in Nash stability); and (ii) what the deviators are entitled to do (for example, form a new, self standing coalition in core stability; join an already existing coalition regardless of how the incumbent members are effected in Nash stability). When it comes to games about the development of hedonistic coalitions, the second point may be investigated by including membership rights. Sertel (1992) presented an abstract framework

for the presentation of four potential membership privileges. The membership rights that are used indicate the set of agents whose consent is required for each given deviation of a subset of players when a hedonic game and a partition are taken into consideration.

When playing a game that involves the development of hedonic coalitions, the primary concern is whether or not there are partitions that are stable in some sense. Core stability principles have received the majority of research attention up to this point. Coalition formation games and lottery-based distribution systems are the two subfields of multiagent systems that we focus on researching. There is a subgenre of cooperative games called coalition building games, and its primary objective is to group players into various alliances. A supply of items can be dispersed among a group of agents using lottery-based distribution systems, which give a way through which this distribution can take place. Even though general case coalition formation games have the potential to mimic a wide variety of fascinating scenarios, the majority of the research that has been done in this field has been on different sub-classes of coalition building games. Our primary interest is in hedonic games, which are a sub-class of coalition formation games based on the assumption that an agent's utility is entirely drawn from the coalition in which they are currently participating [1, 4]. We give findings for one of the new classes of hedonic games that we offer as well as introduce two additional classes of hedonic games. In addition, we offer a novel stability concept with the intention of better characterizing the behavior of agents within the context of certain realistic assumptions. Our research on lottery-based distribution systems is an extension of the work done by Benabbou and colleagues in the past, which was motivated by Singapore's method for allocating public housing. The Housing and Development Board is responsible for the management of more than 70 percent of Singapore's residential property (HDB). This position as the majority operator of residential real estate is used by the Singaporean government to promote ethnic diversity in neighborhoods. This is accomplished through diversity quotas, which limit the percentage of real estate that can be owned by an ethnic group in any given neighbourhood. Here, we describe the findings of an analysis conducted on a streamlined version of Singapore's public housing allocation system.

OBJECTIVE

1. To study the hedonic games stability

HEDONIC GAMES

Hedonic coalition formation games are a powerful tool to model agent behavior in a variety of circumstances. A classic example is the stable marriage problem which seeks to pair off men

and women from two equally-sized groups. In the stable marriage problem, a pair is said to be stable if there is no man-woman pair who both want to leave their assigned partners to form a new pair.

Consider the process of choosing where to live. Previous research on hedonic games has investigated the process of choosing the right roommates or housing units to ensure a stable configuration. We look beyond the individual housing unit to consider the choice of neighbors, perhaps in a setting where students are choosing dormitories/hostels. We envision the partitioning of students into living units (floors, buildings, etc.) as a hedonic game. It is clear that we value our friends’ happiness with the living situation, as we will hear about it from them; our enemies’ happiness could be assumed to also affect how they treat us. (If we stopped there, we would be modeling evaluation as an Altruistic Hedonic Game (AHG) More generally we can also argue that our friends’ friends’ happiness will affect our friends’, and thus indirectly, our own, and that this continues out friendship chains, with decreasing (or at least, non-increasing) effect as we increase the social distance from ourselves. If we were building intranets, a node could evaluate the quality of the local network in terms of the bandwidth to reachable nodes. However, it would also need to take into account the quality of more distant connections, if it hopes to have its packets relayed. There are many other applications in which agents care not only about immediate connections, but also those farther away. We introduce a family of hedonic games that model such broad evaluations of coalitions: The Super Altruistic Hedonic Games.

ANCHORED TEAM FORMATION GAMES

Consider tabletop role playing games (TRPG) such as Dungeons and Dragons. More specifically, consider a university club focused on such games. One of the key challenges such a club must face is how to divide up the club members so everyone can participate in a game. In order to play a TRPG, a group must have both players and a game master, or GM. We propose Anchored Team Formation Games (ATFGs) as a model for such situations. A key focus of our work on ATFGs is the impact of unknowns on coalition stability. Recent work by Barrot, Ota, Sakurai, and Yokoo has investigated the impact of unknowns on FHGs. In their work, an unknown agent is assigned either an epsilon positive or negative utility value depending on whether the agent assessing the unknown is extroverted or introverted. Our interpretation of unknowns is based on work by Konczak and Lang in voting theory where unknowns are assumed to have a true value that is only known ex post facto.

PROXIMAL STABILITY

This work was inspired by experimental work on decentralized, multi-agent coalition formation. In that work, agents have additively separable preferences, and explore a grid world, learning about their preferences for other agents as they encounter them, and ultimately forming coalitions. As other agents join a coalition, subgroups may “bud off” from the group to form their own, new coalition. In that work, the budding agents must increase their utility by forming the new coalition, while the remaining agents’ utilities are not taken into consideration. We describe a stability notion defined by coalitions that do not fracture into smaller coalitions. This contrasts with core stability wherein a deviating coalition can comprise agents from up to as many coalitions as there are agents in the new coalition. It is reasonable to think of this as a scenario where agents have limited information about agents from coalitions other than their own. Our work thus far has focused on distinctions separating proximal stability from core and Nash stability. We have proved that proximal stability neither generalizes or is generalized by Nash stability. We prove that proximal stability generalizes core stability.

LOTTERY-BASED ALLOCATION SYSTEMS

We are interested in modeling allocations of goods over diverse populations similar to HDB housing allocation in Singapore and school selection in several US cities. We construct a lottery-based allocation model that utilizes a set of players N , which is partitioned into k types $\{N_1, \dots, N_k\}$, and a set of goods G , partitioned into l blocks $\{G_1, \dots, G_l\}$. We do not always define a fixed size for N and sometimes define N simply as a random distribution vector. Each agent $i \in N$ has a set of approved goods $G_i \subseteq G$. The utility some agent $i \in N$ derives from some good $j \in G$ is 1 for approved goods and 0 otherwise. Using this allocation model, we investigate whether we can ensure fairness with agent type maximum quotas. Additionally, we characterize the probability that an agent i of type h will not receive a good because of their type as a function of their position in the allocation queue. We investigate one variation on this allocation problem, wherein the set of players N is partitioned into groups of fixed size such that each group’s type composition closely replicates the type composition of N . We utilize a series of simulations to better understand how certain conditions impact the outcomes of such allocation systems. We craft a mathematical model of agents’ expected utility based on their distribution queue position and their type. We use this expected utility model to evaluate the performance of Singapore’s HDB allocation system with 2018 population data and found that

members of one of the three ethnic category have substantially worse expected outcomes than the other two.

According to the membership rights known as free exit-free entry (FX-FE), every agent is permitted to make any movements among the coalitions that make up a given partition without first obtaining the permission of members of the coalitions that she leaves or joins, regardless of which coalitions she moves between. In the context of the roommate issue, an illustration of this would be the provision that an agent is permitted to move into a room as soon as she discovers there is space available in that room. Therefore, it may be beneficial for two agents working in separate rooms to switch rooms without inquiring with anybody else. One such illustration of this is the fact that a citizen of a nation that is a member of the EU is able to relocate to another nation that is a member of the EU without the consent of either nation.

An agent has the right to free exit-approved entry (FX-AE) membership, which allows her to leave her current coalition without the permission of her current partners. However, in order for her to join a new coalition, all of the members of the new coalition must welcome her; in other words, her joining the new coalition must not cause any harm to any of the members of the coalition she joins. A typical illustration of this may be seen in the concept of club membership. A person who is already a member of a club is free to quit that club at any time, regardless of whether or not doing so will negatively affect the other members of the group. Nevertheless, in order for her to join a club, she must first receive consent from its existing members. Another illustration is that of a researcher who is part of a research team but is free to leave the team at any time without the permission of the other members of the team. On the other hand, the researcher's application to join another team is typically subject to the approval of the members already working on that team.

Every agent is granted privileges under the authorized exit-free entrance (AX-FE) policy, according to which she is only permitted to quit her existing coalition if the members of that coalition give their consent to her departure, but entering a new coalition does not require the consent of anyone else. A good illustration of this would be an army that is looking for volunteers. It is possible for any person who is physically fit and falls within a specified age range to enlist in the military if they so want, but once they do so, they are not free to leave the service at any time.

According to the authorized exit-approved entrance (AX-AE) membership rights, any player who quits or enters a coalition is required to have the consent of all of the members of that coalition in order to do so. The example of a criminal organization is a good one to illustrate

this point. Because she could know some of the group's most closely guarded secrets, an undercover spy who is also a member of a criminal organization is not allowed to quit the group without first receiving authorization. In the same vein, it is impossible to become a member of a criminal organization without first receiving approval.

SUPER ALTRUISTIC HEDONIC GAMES

Think about the process of deciding where you want to reside. Let us take into consideration the choice of neighbors, possibly in the context of a scenario in which students are selecting the dorms or hostels that they will reside in. The practice of segregating students into separate housing units (floors, buildings, etc.) might be seen as a hedonistic game. It is obvious that we place a high value on the happiness of our friends with regard to the living condition, as we will hear about it from them; conversely, it is reasonable to expect that the happiness of our adversaries will also impact how they treat us. If we were to leave it at that, we would be portraying assessment as a hedonistic altruistic game. More generally, we can also argue that the happiness of our friends' friends will affect the happiness of our friends, and therefore indirectly, our own, and that this continues through our friendship chains, with the effect decreasing (or at least not increasing) as we increase the social distance from ourselves.

If we were developing intranets, a node would be able to judge the quality of the local network based on the bandwidth available to other nodes in the network. If it is to have any prospect of having its packets relayed, however, it will also need to take into account the quality of connections that are further apart. There are a great number of different applications in which agents worry not only about nearby connections, but also about connections that are further away. The term "Super Altruistic Hedonic Games" refers to a set of hedonic games that we provide here that simulate such comprehensive assessments of coalitions.

AHGs take into account the preferences of their friends, but we feel it is more effective to take into account everyone in the coalition. When this is done, it becomes feasible to represent scenarios that are larger and more complicated than those that can be modelled using AHGs. For instance, we might make use of this to represent an extension of the stable roommate problem to the challenge of distributing all of the rooms on a dormitory floor. Because we are counting each person living on the floor as a part of the same coalition, we will refer to coalition members as residents. members are not tied to one another to the same extent as they are in the conventional roommate situation. Nevertheless, because members of the coalition continue to live in such close proximity to one another, it is in their best interest to get along with other

coalition members.

SAHGs are a natural extension and generalisation of altruistic hedonic games (AHGs, see Definition 17), which are games in which actors take into account the preferences of other agents. In AHGs, agents will solely take into account the preferences of their close companions.

In SAHGs agents give careful consideration to the desires of every other agent in their coalition. A value out of n is given to each buddy in an AHG, where n is the total number of agents. In SAHGs, friends are given a fixed value that cannot be negative, and foes are given a set value that cannot be positive; however, there are no rules that stipulate what those values must be. Additionally, in SAHGs, the preferences of all of the agents that are a part of a coalition are taken into consideration. This frequently involves making use of indirect links, such as friends of friends, in order to modify the weight that is given to the preferences of others. (It is important to remember that friendship is not a one-way street; one of our allies may be betrayed by a friend of theirs.)

While we extend the key conceptions of AHGs to take into account all agents in a coalition rather than just friends, there are additional ways in which the concepts of AHGs might be extended. AHGs were suggested to be expanded into the more general realm of coalition formation games by Kerkmann and Rothe in the year 2020 Their proposal allowed for an agent's altruistic conduct to extend beyond the boundaries of their coalition.

A type of coalition building games known as Social Distance Games (SDGs, see Definition 18) is one in which an agent's utility is a measure of their proximity to, or social distance from, the other members of their coalition.

SDGs and SAHGs share certain similarities, but we believe that SAHGs can more accurately mimic genuine human interactions since they combine the concept of social distance with the consideration of the preferences of others that is presented in AHGs.

SAHGs generalize Friends-oriented and Enemies-oriented Hedonic Games (FOHGs and EOHGs, see Definitions 15 and 16), as we will demonstrate later with Proposition 1 [20]. In the first scenario, agents are tasked with building coalitions with the primary objective of increasing the number of allies while simultaneously working to reduce the number of adversaries. The latter has as its primary objective the reduction of the number of adversaries, while the former has as its secondary objective the increase of the number of allies. This conclusion makes sense given that AHGs primarily relied on the fundamental concepts espoused by FOHGs.

In further study, the ideas of FOHGs and EOHGs are extended in various ways.

1. Ohta et al. research analyses the influence that neutral actors have on these games, where a neutral agent is defined as an individual who is neither a friend nor an enemy. Ohta et al. demonstrate that by allowing neutral agents in EOHGs, it is possible to create games that do not have a core stable partition. Core stable partitions are still guaranteed to exist in FOHGs with neutral agents, but strict-core stable partitions are not. Core stable partitions are still guaranteed to exist in FOHGs with neutral agents.
2. Kerkmann et al. look into games that have friends, foes, and neutrals, and where the friends and enemies are ranked against one another. demonstrate that verification of possible and required stability is in P for individual rationality, Nash stability, individual stability, and contractual individual stability, but it is coNP-complete for core stability, stringent core stability, and strict popularity. Kerkmann et al. found that demonstrating the existence of potentially and necessarily stable partitions is in P for individual rationality and contractual individual stability, while it is NP-complete for Nash stability and coNP-hard for stringent popularity. Determining if there are certainly individually stable partitions exists is an NP-complete problem, but determining whether there are possibly individually stable partitions exists is only known to be an NP problem.

ANCHORED TEAM FORMATION GAMES

Take tabletop role playing games (TRPG) like Dungeons & Dragons for example. To participate in a tabletop roleplaying game (TRPG), a group needs both players and a game manager (GM), with the former tasked with overcoming obstacles posed by the latter. The anticipation of having a good time with a specific group in order to play such a game is predicated on the anticipation that the game master (GM) will set up a good story line, complete with adequate obstacles and rewards, and that the other players will offer good measures of cooperation and competition. We examine the development of groups with a leader, or anchor, by introducing anchored team formation games (ATFGs) and utilising the formation of Tabletop Role Playing Game (TRPG) groups as an example application. Although throughout the article we refer to the gaming application, the anchor might just as easily be a team lead in a programming or engineering team, in business, in class project groups, or even in outdoor adventuring. Regardless of the context, the essential question is: how can we organize the individuals into groups that will successfully do the task (or play the game), and how can we select leaders for the groups in a way that is in line with the preferences of the individuals?

We are interested in finding out whether stable partitions are possible for a certain ATFG, as well as how to locate them if they do exist. Given the use case that we have given, we consider Nash stability to be the most applicable kind of stability that can be found in the literature. Nevertheless, the challenge of locating a Nash stable partition is considered to be NP-hard. In this paper, we report the results of our experiments along with three efficient algorithms for determining whether or not ATFGs include Nash stable partitions. Our first algorithm begins by choosing anchors around which to construct coalitions. Following this step, we split players in a round robin way consisting of numerous rounds, during which each coalition picks one person to add to its ranks. Our second method is a local search implementation that strives to reduce the number of blocking players. These players have the potential to increase their own utility by unilaterally deviating from their assignment, and our goal is to eliminate as many of these individuals as possible. Our third method of calculation uses the results of the round-robin algorithm as a starting point for a local search, so continuing the daisy-chaining process begun in the previous two steps.

Experiments conducted on scenarios including 10–12 agents reveal that all three strategies function admirably well. On the other hand, we see that daisy-chaining provides a substantial improvement over either individual approach when applied to bigger cases. The results of these tests suggest that useful partitions can be discovered. In the following paragraph, we will present a more in-depth justification for doing this job. After that, we provide a formal definition of ATFGs and show the methods that were utilised, the experimental set-up, as well as the findings.

Consider the possibility of a club at your college or institution that is devoted to playing TRPGs. The club has a significant obstacle in dividing up its members so that everyone may take part in a game while also ensuring that everyone is satisfied with the arrangement. It's possible that they'll be able to rapidly estimate how many groups should be established, but it's less likely that they'll have a good idea of who should be placed in each group. On the topic of who would make a good general manager, not everyone will agree. There are a variety of points of view about the kinds of narratives and difficulties that should be included in the greatest games, and these points of view may have a significant influence on the kinds of games that a GM produces for their players or on the kinds of games that players like playing. This indicates that Azar may consider Dawa to be a great GM, whilst Cleo may find Dawa to be insufferable as a GM. This difference in opinion may be attributed to the differences between the kinds of games that Dawa prefers to produce and the kinds of games that Azar and Cleo enjoy to play.

People have preferences about the people they play with as well. Some individuals may find it more enjoyable to compete against other players who are adept at following the plot of the game, while others may find it more enjoyable to compete against other players who are talented at developing their characters' offensive and defensive capabilities. Just as with general managers, not everyone will have the same opinion about who the most enjoyable people to play with are. Consider attending a scholarly conference instead, such as the one hosted by the AAAI, which features breaks that are spaced out among the different sessions held each day. The majority of guests left at each of the breaks.

will interact with one another in a discourse of a more relaxed kind. Attendees have a habit of moving around for a few minutes at the beginning of the break before settling down with one group, with whom they will spend the bulk of the break conversing. This behaviour is rather common. The majority of those in attendance are either students, post-doctoral researchers, or university professors. Both professors and post-docs will normally have some level of recognition within the community of researchers, with professors often having a higher level of recognition than post-docs. In contrast, students will likely be less well-known than faculty members. During these pauses, we suggest that the professors and, to a lesser extent, the post-docs will frequently act as anchor points for conversation groups. As is the case with TRPGs, not everyone will have the same opinion on which researchers are the most qualified to chat with. For instance, many individuals may find it more beneficial to spend their time speaking with other researchers that specialize in themes that are analogous to their own.

RESULTS

We talk about the outcomes of our experiments in this section. Due to the previously mentioned code problem that was found, we have left out the findings from LS testing in the oldest experiment, Randomly Generated Instances: Part 1. The problem was addressed before those tests were conducted, therefore LS findings are included in the trials on the hand-crafted instances and fresh randomly generated instances.

Randomly Generated Instances: Part 1

Our testing of the daisy-chain heuristic employ a factor of 10 fewer restarts for most tests in order to get more comparable runtimes because we see that the daisy-chain heuristic is much slower per iteration than URR.

While identifying Nash stable partitions using URR was not as successful as we had hoped, it frequently produced partitions that were close to stable, or with a low degree of instability. Keep in mind that the amount of agents who would individually choose to depart from their designated coalitions determines the degree of instability. Local search performed noticeably better than each algorithm alone in relocating those people. Refer to Table 1.

For examples of sizes 25 and 30, we see that the proportion of cases for which the heuristics discovered Nash stable partitions is quite low. With more cores on our VMs, we are optimistic that the algorithms will run noticeably faster, allowing for more restarts, which should raise these percentages. We ascribe this to a lack of random restarts. We have not yet been able to confirm the

the existence of Nash stable partitions on those examples using the full procedure, but there is a strong likelihood that a higher proportion of them do than our results suggest.

Table 1 shows the typical runtime (in CPU seconds) and the proportion of situations where NS coalitions are discovered.

# Players	URR		DC	
	time	%	time	%
10	24.5	47.3	2.0	36.5
12	43.2	17	22.9	52.5
15	239.0	65.5	44.6	71.5
20	898.4	20.4	178.1	25
24	268.4	94.4	748.5	79.6
25	1745.7	0	591.9	4
30	2605.8	7.6	1533.8	2.5

Note: Fifty thousand restarts were permitted for the 24 agent DC test.

The number of stabilizable instances among the randomly generated ones is as follows: 48% of our 10-player instances, 60% of our 12-player instances, 95% of our 15-player instances, and 96% of our 20-player instances. Because the utility ranges for the 10 and 12-player instances are both -20 to +20 (i.e. larger than the number of agents), as opposed to the other instances, which were generated with limits no wider than -n to +n where n is the number of agents, our initial hypothesis was that a smaller percentage of the 10 and 12-player instances are stabilizable. This notion appears to be false after looking at freshly formed 10-player

instances in- tended to be used for new trials. Future research will focus on factors that affect how likely it is for a specific instance to be Nash stabilizable.

We offer data on the number of agents who desire to relocate, the degree of instability (DoI), and the coalitions that DC and URR have identified. This offers more proof that DC typically outperforms URR in terms of performance.

Table 2: Average DoI

# Players	Number of movers	
	URR	DC
15	0.345	0.045
20	0.952	0.116
24	0.056	0.028
25	2.455	1.105
30	1.3	1.18

HAND-CRAFTED INSTANCES

Table 3's summary of the findings from our tests on the hand-crafted cases demonstrates that local search performs substantially worse than either of the other heuristics at locating stable partitions. For the majority of these situations, the number of possible partitions is substantially bigger than the number of Nash stable partitions, so LS must be incredibly fortunate to start close to a local optimum that is actually globally optimal. This is demonstrated by its subpar performance in comparison. However, we observe a significant improvement in runtime and the quantity of restarts required to discover a stable partition when local search is applied to the round robin heuristic's output.

Table 3: The percentage of tests that contain NS coalitions, the average runtime (in CPU seconds), and the average restarts.

# Players	URR			DC			LS		
	time	restarts	%	time	restarts	%	time	restarts	%
10	0.10	111.9	100	0.02	5.1	100	41.8	9001	10
10	0.08	81.1	100	0.03	11.5	100	19.8	8001	20
25	7.4	2210.2	100	0.01	1	100	210.9	5005	50

Note: At least one known Nash stable partition can be found in each hand-crafted instance. On every experiment, all three heuristics arrived at partitions with a DoI of 0. This appears to be at odds with how quickly LS was able to identify Nash stable partitions, however many of the local search results had singleton agents who were unwilling to join any of the available coalitions. Such partitions are invalid and cannot be Nash stable since singletons are not valid coalitions for any of the hand-crafted instances. On the other hand, URR and DC immediately discovered Nash stable partitions on all three cases.

CONCLUSION

In this study, two brand-new kinds of hedonic games and a fresh definition of hedonic game stability are presented. We have shown that hedonic games have several uses and can mimic a variety of social phenomena. In the area of hedonic games, we have provided answers to a number of questions, but there are still more that need clarification or that haven't yet been thoroughly established. The space of hedonic games contains several computationally challenging challenges. Fortunately, heuristics can often address many of these challenging issues; this was made abundantly evident by our work on the Anchored Team Formation Games. Many issues in the field are still unresolved. Can we, for instance, develop effective heuristics to examine internal stability? If so, how will they differ between different types of hedonic games? We've shown that internal stability for ATFGs differs from both Nash and core stability, but are there more stability concepts that exist along this range of local to global scope? If so, are these ideas continuous or discrete in nature?

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